Ultra-sensitivity of numerical landscape evolution models to their initial conditions

Jeffrey Kwang[1] and Gary Parker[1,2]
[1]Department of Civil and Environmental Engineering, University of Illinois at Urbana-Champaign
[2]Department of Geology, University of Illinois at Urbana-Champaign

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Steady State Behaviors

Landscape evolution models (LEMs) can obtain two types of steady-states, a flux-based steady-state or a topographic steady-state (see Willett and Brandon, 2002).

- **Flux-based steady-state (FBSS)**: total influx of material into the control volume (via uplift or base-level fall) equals total outflux of material (via erosion).
- **Topographic steady-state (TSS)**: local incision and uplift are in balance in all locations, or incision is spatially uniform in the case of a lowering base-level. This produces a "frozen" landscape.

Willett, S. D., & Brandon, M. T. (2002). On steady states in mountain belts. Geology, 30(2), 175. https://doi.org/10.1130/0091-7613(2002)030<0175:OSSIMB>2.0.CO;2

Numerical Landscape Evolution Models

$$\partial \eta / \partial t = -KP^m A^m S^n + D\nabla^2 \eta$$

$$\partial \eta / \partial t \,|_{y=0} = -B$$

η - elevation; t - time; K - erodibility constant; P - precipitation rate; A - drainage area; S - slope; m,n - positive exponents; D - hillslope diffusion coefficient; x,y - horizontal coordinates; B - base-level lowering rate

- Numerical LEMs are sensitive to their initial conditions because they are deterministic.
- Under constant forcing, landscapes in numerical LEMs tend towards TSS.
- A common initial condition consists of a horizontal plane with randomized topographic perturbations.
- Boundary conditions are shown in red below and remain the same throughout this presentation.

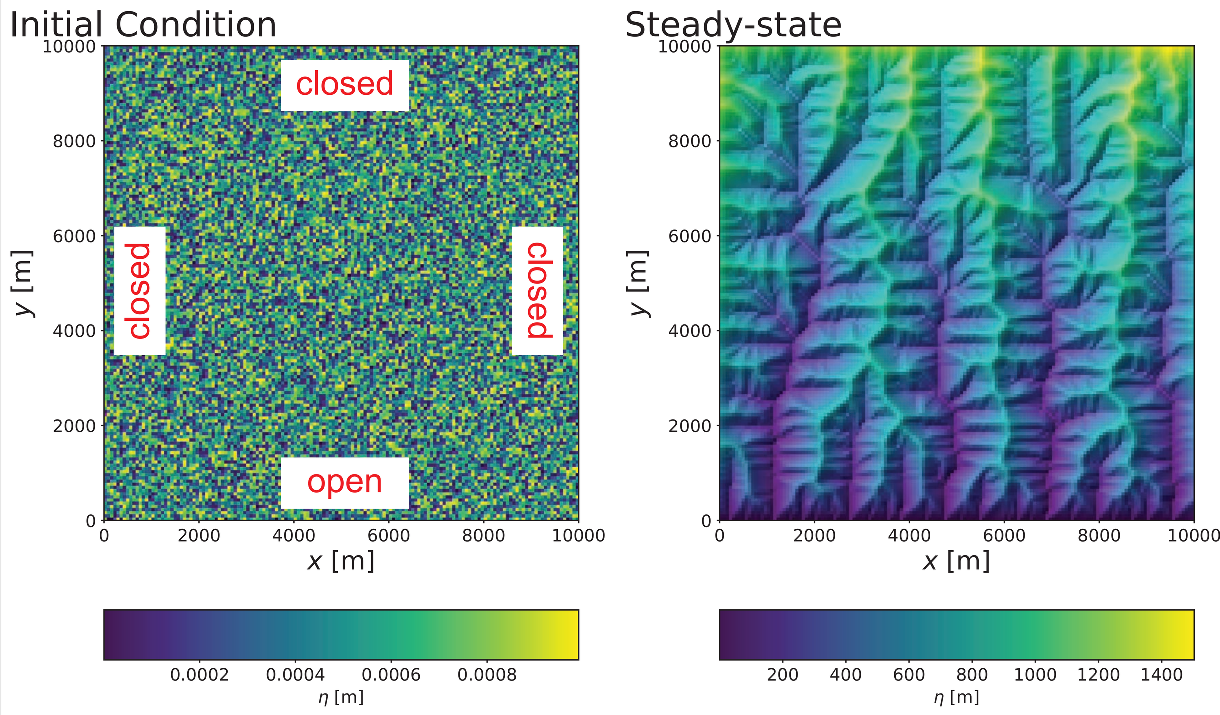

Figure 1: An initial randomized topography evolves into a TSS landscape. Here, $K = 1\times10^{-5}$ $m^{-0.5}$ $yr^{-0.5}$; $P = 1$ m yr^{-1}; $m = 0.5$; $n = 1.0$; $D = 0.03$ m^2 yr^{-1}; $B = 1\times10^{-3}$ m yr^{-1}

Ultra-sensitivity

- Numerical landscapes retain many topological features and signals from their initial conditions.
- **Ultra-sensitivity** describes the numerical model's ability to preserve minute perturbations from the initial topography.

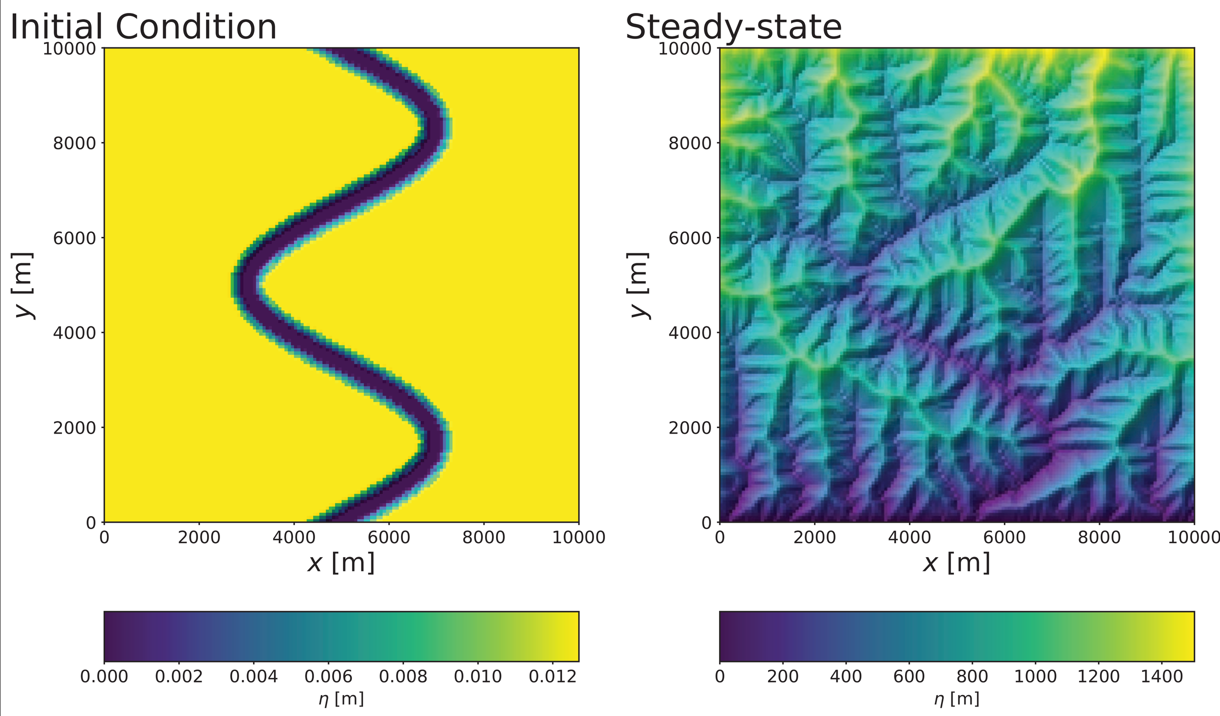

Figure 2: A shallow sinusoidal channel in the initial topography evolves into a deep canyon at TSS. Here, $K = 1\times10^{-5}$ $m^{-0.5}$ $yr^{-0.5}$; $P = 1$ m yr^{-1}; $m = 0.5$; $n = 1.0$; $D = 0.03$ m^2 yr^{-1}; $B = 1\times10^{-3}$ m yr^{-1}

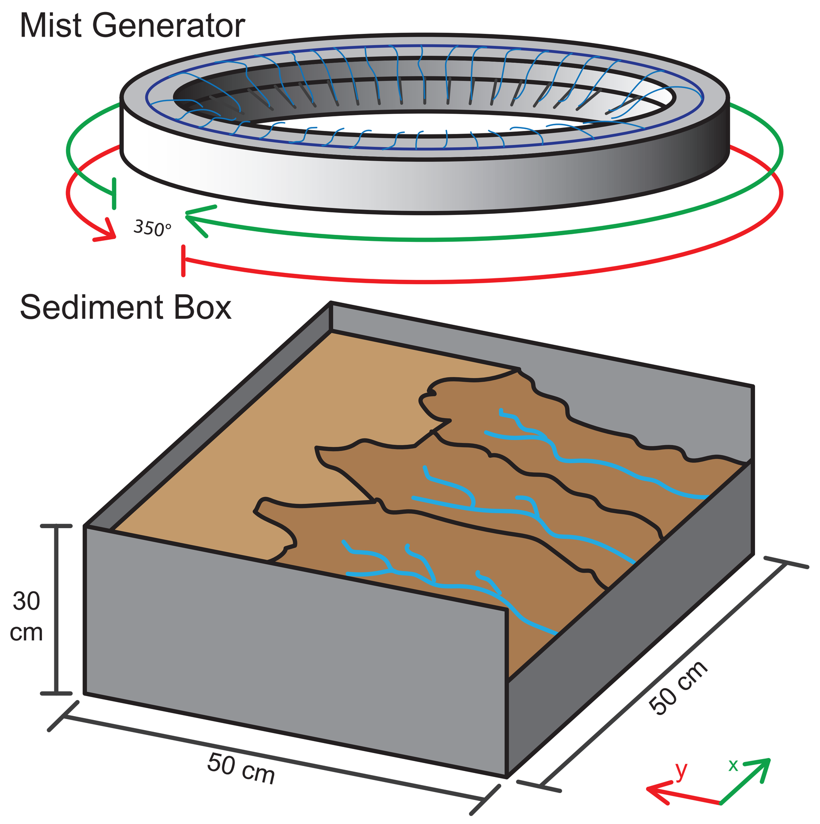

Figure 3: XLE facility schematic

eXperimental Landscape Evolution (XLE) Facility

- Mist generator (P range: 16 mm hr^{-1} - 411 mm hr^{-1})
- Two independently movable weirs (B range: 5 mm hr^{-1} - 200 mm hr^{-1})
- Takes planform images and generates digital elevation models (DEMs) at a user-specified frequency (12 hr^{-1})
- DEM resolution = 0.5 mm
- Experimental substrate is made from a silica flour (grain size ≈ 23 μm) water mixture (65:35 ratio)

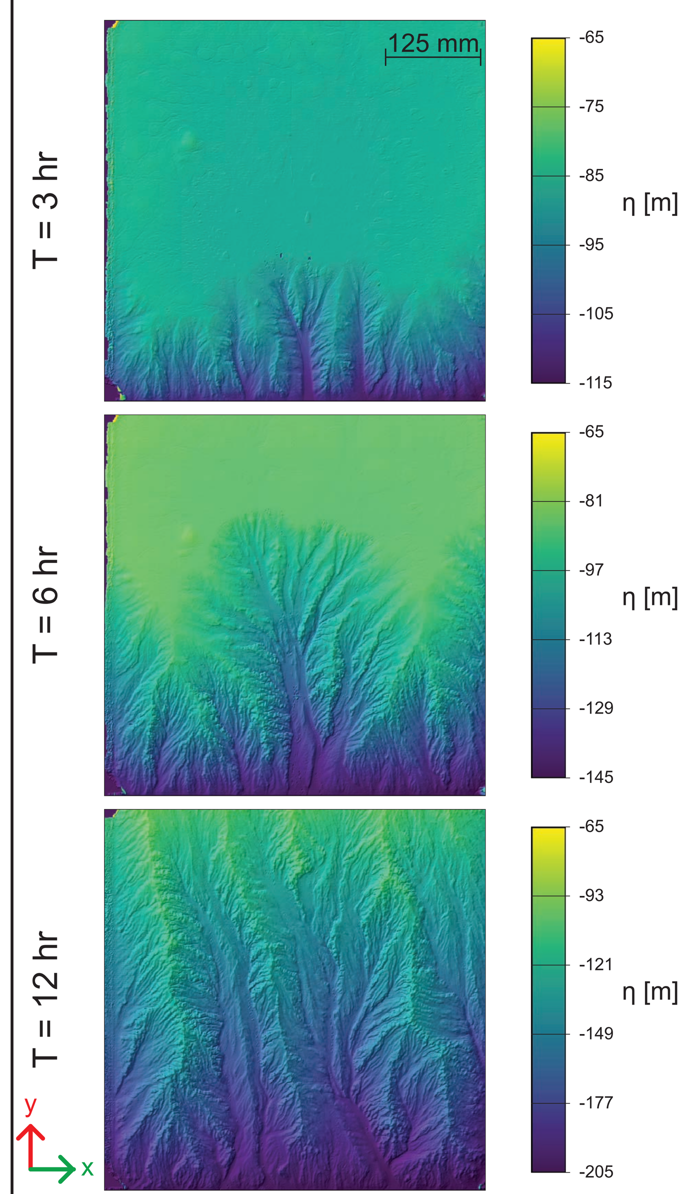

Figure 4: Time evolution of the control experiment (3, 6, 12 hr)

Control Experiment

We use a control experiment to calibrate parameters for our numerical model.

Parameters that are directly measured:

- $B = 10.185$ +/- 0.005 mm hr^{-1}
- $P = 38$ +/- 4 mm hr^{-1}

Parameters that are fitted to the DEMs:

- $D = 7$ (+6/-2) mm^2 hr^{-1}
- $K = 0.97$ +/- 0.05 $mm^{-0.011}$ $hr^{-0.663}$
- $m = 0.337$ +/- 0.006
- $n = 1.0$ (assumed)

In the experiments, **channels shift laterally** and **autogenically generated knickpoints migrate upstream** throughout the landscape's evolution. These processes are absent from the general numerical model's formulation.

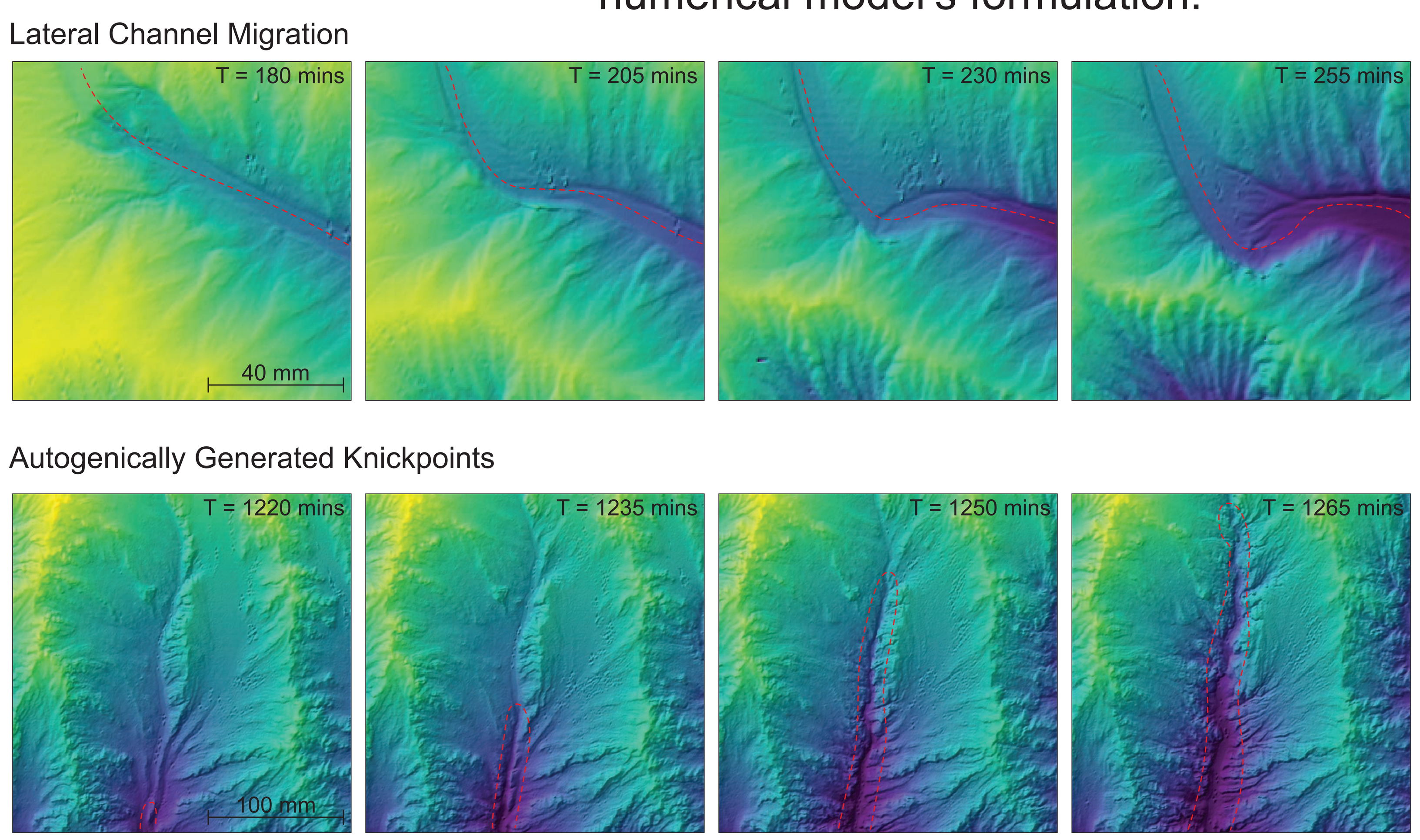

Figure 5: Time evolution of lateral channel migration (top row) and autogenically generated knickpoints (bottom row) in the XLE facility

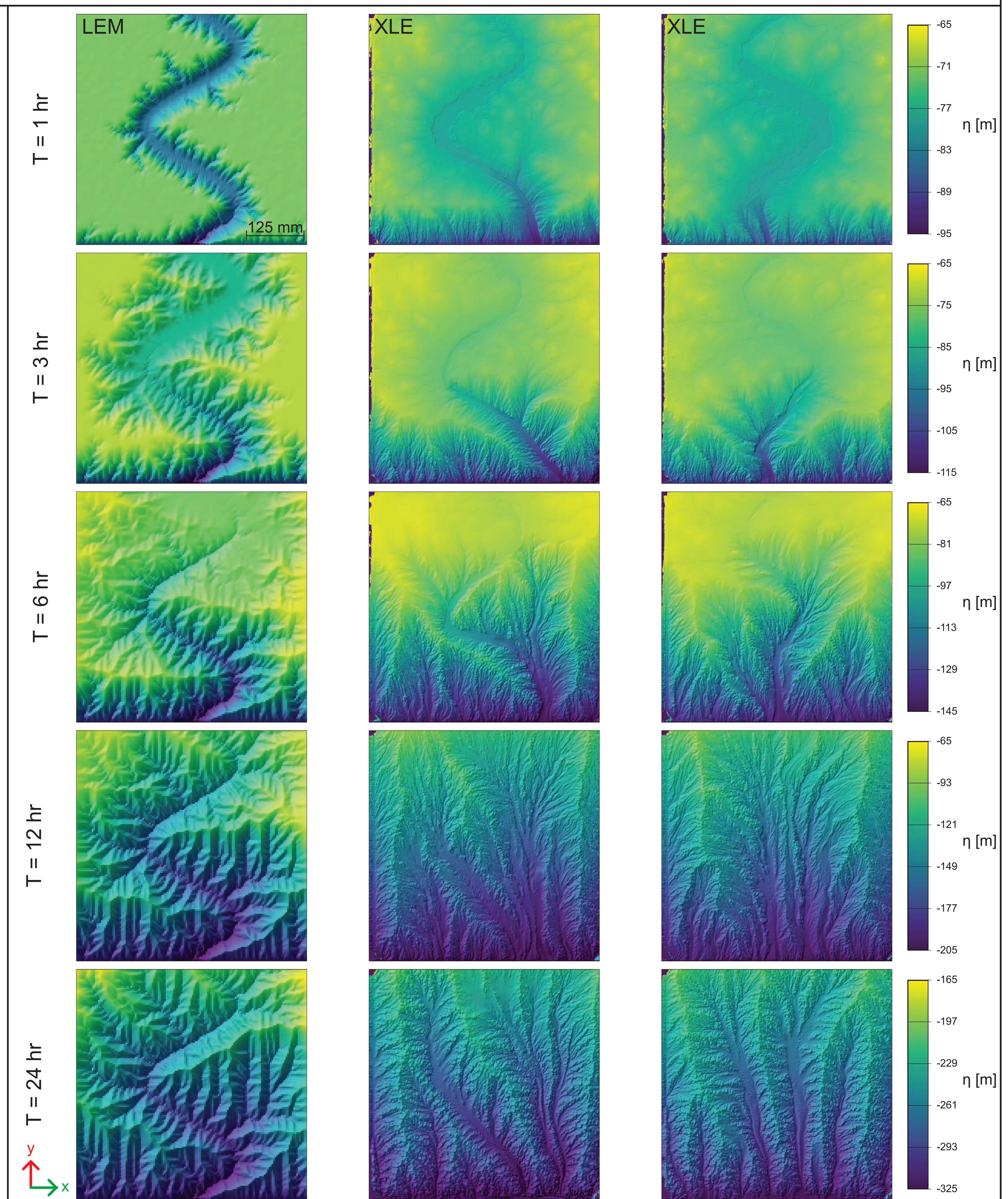

Figure 6: 24 hr landscape evolution of the numerical model (left column) and the XLE experiments (center and right columns). The sinusoidal signal is preserved throughout the landscape evolution in the numerical model but is erased in the experiments.

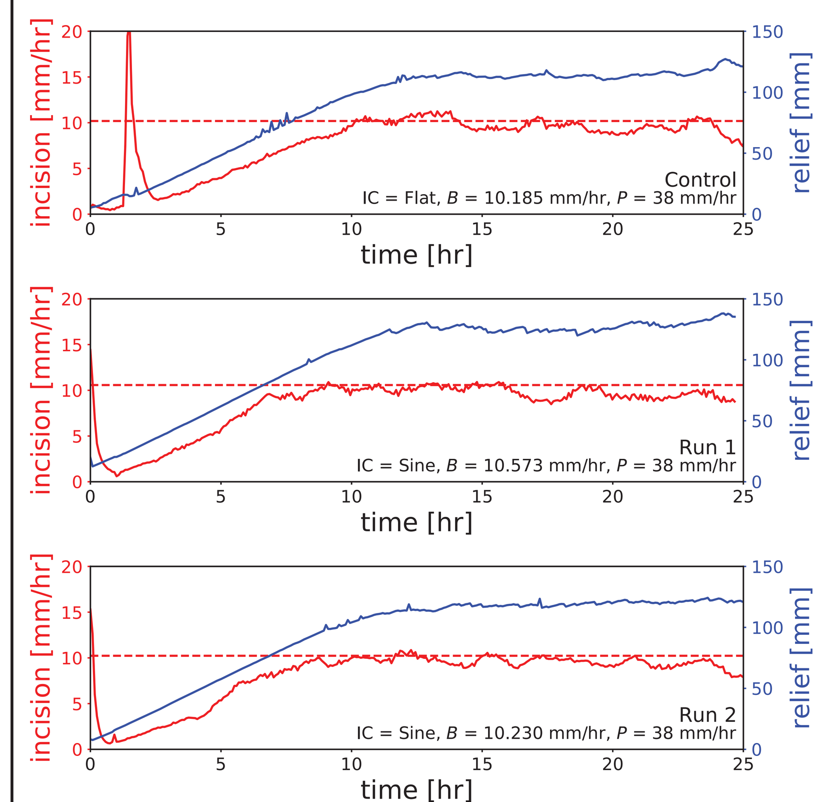

Figure 7: Time series of mean incision rate and total relief in the XLE experiments.

Numerical Model vs. XLE Experiments: Conclusions

- Numerical models preserve small perturbations from their initial conditions indefinitely after achieving TSS.
- Information in the initial conditions of experimental landscapes degrades over time, which is likely due to lateral channel incision and spatio-temporal fluctuations in incision, which are absent from general numerical LEMs.
- Our experimental landscapes achieve FBSS but not TSS.

Acknowledgments

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Model Data